Correction of Radial Distortion using a Planar Checkerboard Pattern and its Image

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Abstract — The images captured by real cameras often suffer from lens distortion which is a nonlinear and radial distortion. In this work, we present a method to correct the radial distortion caused by camera lens. In particular, we discuss a framework to determine distortion vectors and distortion function directly without employing approximation approach which often causes incorrect warping problem. The proposed framework is entirely non-iterative, and therefore is computationally efficient. We also compute the center of distortion, which is important to obtain optimal results. The accuracy and robustness of the proposed system are verified on the experimental results of several indoor and outdoor images.

Index Terms — Lens distortion, radial distortion, image warping.

I. INTRODUCTION

The image generation of any kind of camera is usually modeled with the linear pin-hole camera model. However, the images of real cameras suffer from lens distortion, which is a nonlinear and generally radial distortion. The radial distortion makes straight lines appear curved in the photograph and deforms the captured image nonlinearly. In practice, this problem is more significant in wide-angle cameras. The wide-angle cameras are quite useful in many applications because they provide a large field of view and more visual information from a single image. But they severely bend the optical rays, and the projective mapping between real world scene and imaged 2D point is more complicated than the traditional linear model. For the use of images such as architecture photography, camera calibration, or stitching of panoramic images, the radial distortion often causes critical problems. The cure for this distortion is to correct the image measurements to those that would have been obtained under a perfect linear camera action so that the camera projection becomes the ideal linear pin-hole model. In this paper, we describe an automatic approach to correct the radial distortion. The proposed method relies on the use of a planar checkerboard pattern, which is often used for the conventional camera calibration technique. Our method employs the theoretical basis presented in the previous works[1-3]. However, we discuss a scheme to estimate a distortion function directly, while the previous method approximates it via least-squares estimation. Specifically, the previous method approximates a planar projective mapping between a captured image and the known (reference) features using the least-squares estimation technique. Once the projective transformation is given, then it is possible to compute a set of geometric disparities caused by the radial distortion. Then the distortion function is computed from the disparities, and this is the typical procedure of the previous radial distortion correction. However, one problem is that the estimation of distortion function is seriously affected by approximation error which is due to the scale ambiguity of the projective camera model. Assuming an exact approximation, we get an undistorted image that would have been captured under the linear camera projection. In practice, however, it is not so easy to obtain accurate result in the presence of the approximation error. In such case, the final result is incorrectly warped image in which the lens distortion is not completely removed. In order to minimize this problem, we present a method to determine the distortion vectors and distortion function directly. In the proposed method, the distortion correction includes the following steps. First, the center and the directions of distortion are estimated from a point-to-line projective mapping between the known feature points on the checkerboard and their images. Next, the distortion vectors which move the ideal points to the distorted positions are computed in the affine space. The distortion vectors are then aligned along the direction of expansion away from (or toward) the distortion center so that they have same radial direction (i.e., centrifugal or centripetal direction.) The aligned distortion vectors are transformed back into the projective space, and the imaged feature points are corrected to the undistorted points by the distortion vectors. From the disparities between the distorted points and their corrected

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positions, we fit a quartic polynomial and obtain a distortion function, which is used for the distortion correction. The proposed system has the following advantages. As the distortion vectors and distortion function are obtained directly, the proposed method is more robust to the approximation error. In addition, the distortion correction is carried out via a non-iterative process, and therefore, the proposed method is fast and computationally efficient. We also compute the center of distortion, which is important to obtain optimal results. The accuracy and robustness of the proposed system are verified on the experimental results of several indoor and outdoor images. The remainder of this paper is structured in the following way: In Section 2, the theoretical background is presented, and the framework for the direct estimation of distortion vectors and distortion function is discussed in Section 3. The experimental results are given in Section 4, and we draw the conclusion of the proposed work in Section 5.

II. THEORETICAL BACKGROUND

A. Estimation of direction and center of distortion

The proposed method for radial distortion correction involves the use of a checkerboard pattern as shown in Fig.1(a), on which are feature points with known locations. Assuming an ideal projection, points on the checkerboard pattern would be mapped to undistorted points in the 2D image under perspective camera geometry by a planar homography. In reality, however, we can not obtain these ideal points in the presence of radial distortion. It can be assumed that the imaged points are each moved from their undistorted initial position to the distorted position by expansion away from a center of distortion. In such case, the direction and the center of the expansion can be estimated from a point-to-line projective mapping between the ideal feature points and their images. This situation is analogous to the estimation of epipolar line and epipole in two-view geometry where the camera moves forward toward the scene. The epipole corresponds to the center of distortion, and the epipolar line the direction of distortion. Assuming that the distortion is radial, the distortion can be modeled as

\[ x_i^r = e + \lambda_i (x_i^d - e) \quad x_i^d = Hx_i^r. \]  

(1)

Here \( e \) denotes distortion center, and \( \lambda_i \) is the variable which depends on distortion ratio. \( x_i^r \) represents \( i \)-th distorted point, and \( x_i^d \) is imaged ideal (undistorted) feature point which is mapped by the planar homography \( H \). \( x_i^r \) is original feature point on the calibration grid. If the camera projection mapping is ideal linear projection, \( x_i^r \) will be projected to \( x_i^d \). After multiplying (1) by \([e]x\) which is a skew-symmetric matrix of \( e \), we have

\[ [e].x^r = \lambda_i [e].Hx^r. \]

(2)

After multiplying (2) by \( x_i^{dT} \), we obtain

\[ 0 = \lambda_i x^r ([e].H)x_i = \lambda_i x^r [e].x_i^r. \]

(3)

Eq.(3) represents that the captured feature \( x_i^r \) is mapped to a point of the line which passes through the ideal feature \( x_i^r \) and the center of distortion \( e \). Here, we can define a projective mapping between \( x_i^r \) and \( x_i^d = Hx_i^r \), and this point-to-line mapping may be represented by \( F=[e].H \). The \( 3 \times 3 \) matrix \( F \) of rank 2 is called as the fundamental matrix for radial distortion[1]. Once \( F \) is obtained, then we can estimate the center of distortion from its left null vector (i.e., left epipole), and the direction of distortion is given as the epipolar line corresponding to each \( x_i^r \), as follows.

\[ e^T F = 0, \quad Fx_i^r = 1^T. \]

(4)

B. Estimation of the homography between the calibration grid and its image

If the projective mapping between the calibration grid and its image is known, then it will be possible to compute the distortion function which moves the ideal feature \( x_i^r \) to the captured feature point \( x_i^r \). Assuming that the center of distortion coincides with the image center, that is \( e = (0, 0, 1)^T \) in the homogeneous coordinate, the homography is formulated as \( H = [f_1^T; -f_2^T; 0]^T \). Moreover, from \( F = [e].H = [e](H+ev^T) \), we may write the homography as

\[ H = \hat{H} + ev^T = [\hat{H}; v^T] = [f_1^T; -f_2^T; \hat{v}] \].

(5)

where \( v \) means an arbitrary \( 3 \times 1 \) vector, and the semicolon represents that rows of a matrix is stacked on the top of each other. In Eq.(5), \( f_1 \) and \( f_2 \) are already given, and only the third row \( v \) of \( H \) is unknown. From the above relationships, we get the following equation.

\[ x_i^r = Hx_i^r = [\hat{H}x_i^r; v^T x_i^r] = (\hat{x}_i^r; \hat{v}^T)/(v^T x_i^r) = \hat{x}_i^r/(v^T x_i^r). \]

(6)

Here, we define radii of distorted and undistorted points as \( r^d = |x_i^d| \) and \( r^r = |\hat{x}_i^r/(v^T x_i^r)| \), respectively. If the vector \( v \) is correct, then the points \((r^d, r^r)\) would lie along a monotonic curve. In order to obtain the vector which satisfies the monotonic condition, we can find \( v \) minimizing the following equation referred to as the total squared variation [1].
We rearrange Eq.(7) in the form of $AV$, where $V$ is a vector containing all the elements of the $v$, and $A$ is the matrix of coefficients, which may be constructed from the known values of all the $\hat{r}_i^r$ and $r_i^r$. Then, this problem converges to find $V$ which minimizes $\|AV\|$ subject to the condition $\|V\| = 1$. Once the $v$ is given, it is then possible to compute the distortion function which is used for correcting the distorted image.

$$r_i^r = r_i^v = \hat{r}_i^v / (V_i^\top x_i^r).$$

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$$X_i = H_i \hat{x}_i^r,$$  \hspace{1cm} (9)
which determines the direction of distortion vectors, so that they have uniform direction (i.e., negative direction toward the distortion center). From the distorted points and their corrected positions, we can fit a quartic polynomial and obtain a distortion function, as shown in Fig.4. Once the center of distortion and the distortion function are identified, it is then possible to correct the captured images so that the camera projection becomes the linear pin-hole model. Fig. 3(a) shows the feature points transformed into the affine space by \( \mathbf{H} \), and Fig. 3 (b) is an example which shows that the distortion vectors are aligned by \( \lambda_{\text{mai}} \) so that they have uniform direction (i.e., negative direction which means centripetal.) Fig. 3(c) and Fig. 3(d) illustrate the distortion vectors before and after the radial direction alignment, respectively. Fig. 4 shows estimated distortion curves. As described above, the direction of the distortion can be defined along either of inward or outward direction as shown in Fig. 4(a) and Fig. 4(b) respectively. Note that either of the two results can be used to correct input image. Fig.5 presents examples for the image correction. Fig.5(a) shows estimated distortion direction and center and (b), (c) illustrate correction results obtained by warping the input image centrifugally, and centripetally, respectively.

**IV. EXPERIMENTAL RESULTS**

To evaluate the proposed system with respect to the accuracy and robustness, a series of experiments are conducted. Fig. 6 shows the correction results of the proposed method for several pictures taken by different cameras in different scenes. From the results, it is easily verified that the proposed method can reduce the lens distortion and fairly improve the given input images. For the comparison with the previous approach, we illustrate some comparative results in Fig.7. Even though we can obtain good results from the previous method, it may be considerably affected by the approximation error and the lens distortion is not completely removed as shown in Fig.7(c) and Fig.7(d). However, we verify that the proposed system is robust to the problems caused by the approximation error, and gives more stable results than the previous method as illustrated in Fig.7(e) and Fig.7(f). In order to evaluate the proposed method for more severe lens distortion, we also applied the proposed technique to cameras with fish-eye lens. Fig.8 and Fig.9 present the distortion removal results for fish-eye lens which has a field of view of about 180°.
In correcting the fish-eye images, we have no problem with the computation of the distortion center, or the estimation of $H_A$. However, there is a difficulty with the estimation of the distortion function, since it may approach infinity due to some of the feature points near the image boundary. In fact, in estimating the distortion function, it is not necessary to use all the feature points. We may remove points near the boundary of the image in estimating the distortion function. From the corrected results of Fig.8, Fig.9 we verify that the proposed method is good enough to handle with severe radial distortion.

**V. CONCLUSION AND FUTURE WORK**

We discussed a method to correct the radial distortion caused by camera lens. In order to prevent the incorrect warping problem, we proposed a method to estimate the distortion vectors, and distortion function directly without any
approximation. The experimental results verify that the proposed method is good enough to handle with the lens distortion problem. We believe that the proposed scheme can be employed as a useful tool in the camera-lens related works. As a future work, we will conduct further study to employ the proposed method for a real-time imaging system.

REFERENCES


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